Adversarial Reinforcement Learning

for Cyber-Attack Prevention, Detection, and Mitigation

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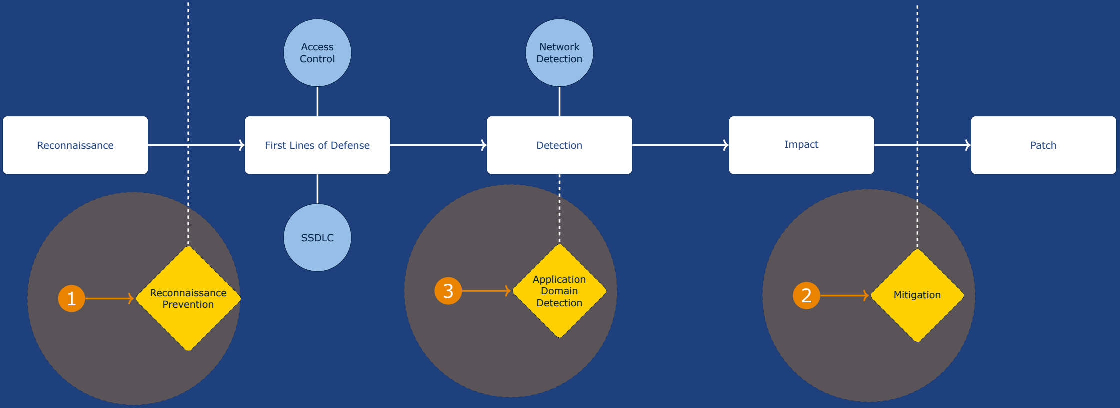
Cyber-Attack Life-Cycle



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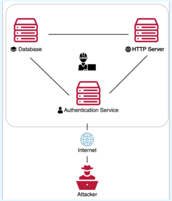
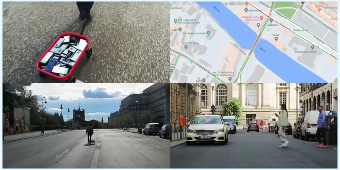
Domain-Specific Cybersecurity Challenges

Moving Target Defense



Mitigation of 0-stealthy FDI

FDI Detection in Navigation



Domain-Specific Cybersecurity Challenges

Moving Target Defense

- MTD is proactive defense.
- Prevents reconnaissance by randomizing configurations, e.g., IP change.
- Technicians decides on when and where.
- Physical connectivity constraints.
- Security vs. efficiency trade-off.

Mitigation of 0-stealthy FDI

- Insider threats are undetectable by network intrusion detection systems.
- Change the **actuation and observation signals** in the operational domain to avoid detection.
- Make **operational adjustments**.
- Example: Stuxnet

FDI Detection in Navigation

- False Data Injection in Navigation application (e.g., Google Maps).
- FDI is happening in the crowdsourced data sources.
- Fails network intrusion checks.
- Must detect system deviations in operational domain.

The Core Problem

Current manual defensive actions are infeasible due to: Dynamic and Non-Stationary Adversaries, Information Asymmetry, Massive Scale, and Intricate Underlying Dynamics and Constraints.

Reinforcement Learning: The Adaptive Solution

Sequential Decision Making Under Uncertainty

- Cybersecurity is not a one-off classification problem; it is a continuous and evolving interaction.
- These domain-specific security challenges require optimal decisions under uncertainty.
- The security challenges require satisfaction of physical and operational constrains, e.g. network connectivity.

RL for MTD

- *Determines the optimal timing for asset randomization.*
- *Considers physical constraints such as network connectivity.*

RL for Resilient Control

- *Minimizes potential damage during an active attack sensor/actuator.*
- *Selects the best actuation signals in uncertain environments.*
- *Considers the safety and operational viability of the system.*

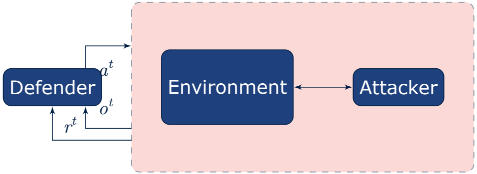
Detection of FDI in Navigation

- *Understands the connectivity of the transportation network.*
- *Learns nominal conditions to identify false data injections.*
- *Maximizes detection accuracy in the application domain while reducing false positive alerts.*

Reinforcement Learning I

Reinforcement Learning (RL) is a Sequential Decision-Making Algorithm **under uncertainty**

- Relies on trial-and-error and dynamic programming to achieve **Optimal Sequential Decisions**
- Learns from *experiences* to maximize expected **Reward**



Reinforcement Learning II

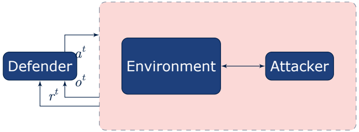
Reinforcement Learning Optimization Problem

Policy or **Strategy** Function

$$\pi(o^t) \mapsto a^t$$

Optimization Problem

$$\max \mathbb{E}\left[\sum_{\tau=0}^{\infty} \gamma^{\tau} \cdot r^{t+\tau} \;\middle|\; \pi\right]$$



In English Please?

*The **policy** function gets the current observation and suggests an action that maximizes "discounted future rewards"*

Discount Factor γ

$\gamma \in [0, 1)$ *prioritizes rewards received in the current time step over future rewards.*

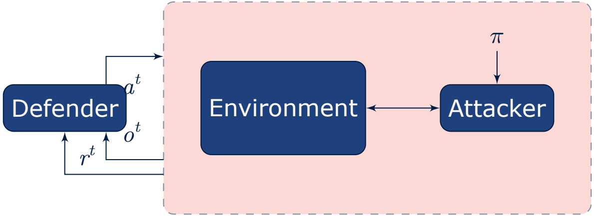
- $\gamma = 0$: *Only care about the current reward*
- $\gamma = 1$: *All future rewards are equal*

Research Questions

RQ 1 How to model the interactions between the **attacker** and **defender** within the application **environment**?

RQ 2 What Reinforcement Learning **algorithm** should we use or develop?

RQ 3 How can we learn if the attacker is **adaptive** and responds to the defender?



Research Approach

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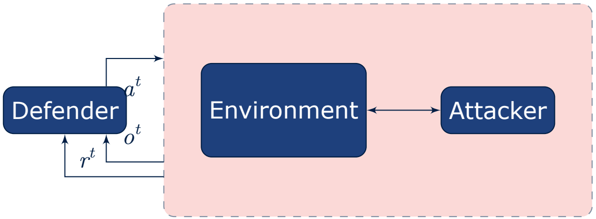
The Interactions

For each of the **Prevention**, **Detection**, and **Mitigation** scenarios, we need to formalize the interactions between the attacker, environment, and the defender.

- What is the **environment model**?
- What is the **threat model**?
- What is the **defender model**?



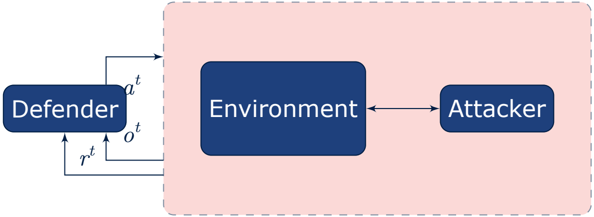
Reinforcement Learning Algorithms

Deep-Q-Networks (DQN)

- **Deterministic** Policy
- **Discrete** Action
- Continuous or Discrete Observation

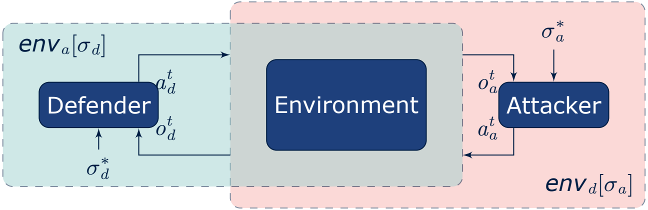
Deep Deterministic Policy Gradients (DDPG)

- **Deterministic** Policy
- **Continuous** Action
- Continuous or Discrete Observation



But, what if the attacker is adaptive?

- An **adaptive** attacker is not static. It has its own **policy** and adapts it to the defender
- The defender must also adapt to the attacker



What is that σ?

That is a mixed strategy: a probability distribution over policies $\boldsymbol{\pi}$.

Two Player Security Game

- The *interactions* and *objectives* of players is expressed as a **Multi-Agent Partially Observable Markov Decision Process (MAPOMDP)**.
- The *parameter selection* for the policy function of each agent, forms a **Normal-Form Game (NFG).**

MAPOMDP

$$\langle P, \mathcal{S}, \{\mathcal{A}_p\}_{p\in P}, \mathcal{T}, \{\mathcal{R}_p\}_{p\in P}, \{\mathcal{O}_p\}_{p\in P}\rangle$$

What are these symbols?

P *set of* ***players***

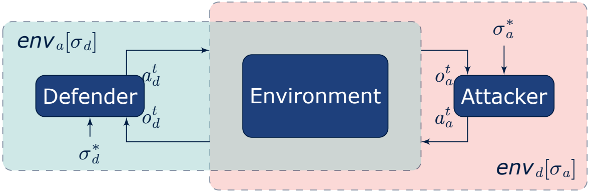
$\mathcal{S}$ *set of* ***states***

$\mathcal{A}_p$ ***action space*** *of player* p

$\mathcal{T}(s, \mathbf{a})$ ***state transitions*** *rules*

$\mathcal{R}_p(s, \mathbf{a})$ ***rewarding*** *rule for player* p

$\mathcal{O}_p$ ***observation*** *rule for player* p



Finding Equilibrium of a Security Game

- We defined a two-player game between the attacker and the defender as a MAPOMDP.
- We assume both players are rational. They always choose a **best-response strategy**.
- This is equivalent to finding the Nash Equilibrium of the security game.

What is Mixed-Strategy Nash Equilibrium?

All players are playing with their best-response to all opponents' strategies, i.e., neither player can increase their expected utility without having their opponents change their strategy.

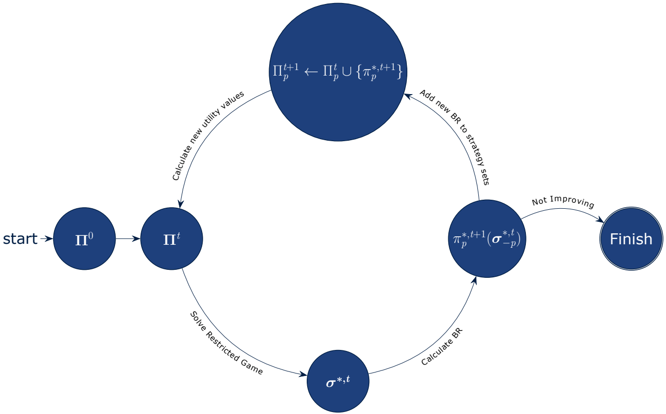
Problem

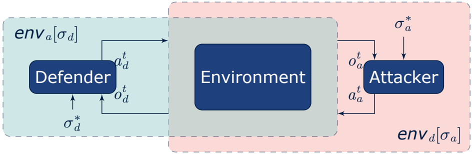
Enumerating all the different strategies to find the Nash Equilibrium is **infeasible**.

Reinforcement Learning as Best-Response Oracle

Reinforcement Learning algorithms can be used to find an *approximate* Best-Response.

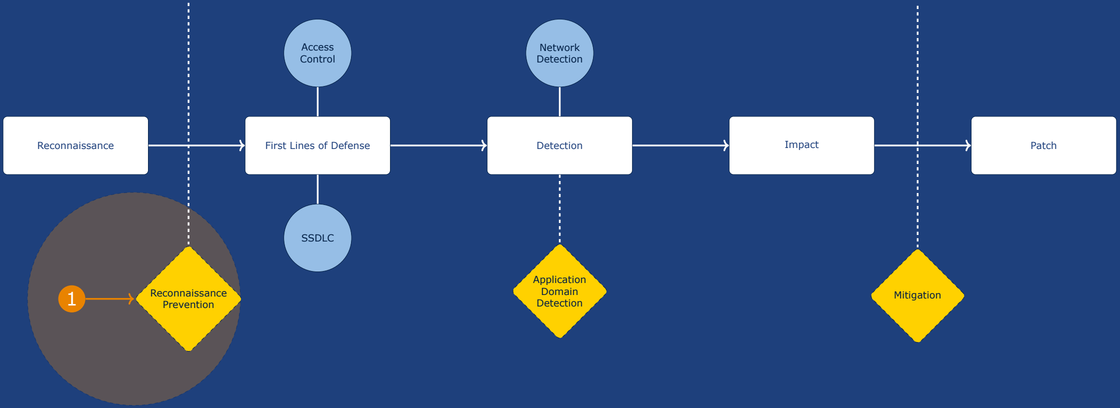
Policy Space Response Oracles





PSRO algorithm [Lanctot et al., 2017] based on Double Oracles [McMahan et al., 2003].

Attack Prevention through Moving Target Defense [Eghtesad et al., 2020], GameSec'20



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The Moving Target Defense Game I

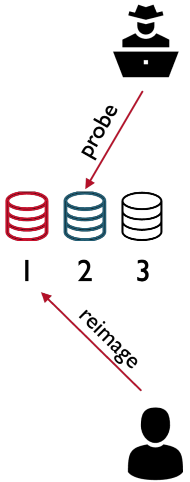
Overview

The Game [Prakash and Wellman, 2015]

An Attacker and a Defender compete over a set of servers.

- Adversary **probes** a server
- Adversary compromises the server with some probability, or
- Increases the chance of compromising that server in the future.

- Defender **reimages** a server.
- Takes the server down for fixed time steps.
- Resets the adversary’s progress on that server
- Takes back the control of the server



The Moving Target Defense Game II

States, Objectives, and Rewards

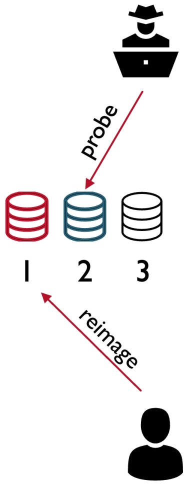
State per each Server

- Number of Probes
- Control
- Up or Time to up

Rewards

Each player is rewarded based on the portion of servers that are in control or down.

- Implicit defender cost, *i.e.*, **not gaining reward** when server is down
- Explicit attacker reward penalty for probing.



The Moving Target Defense Game III

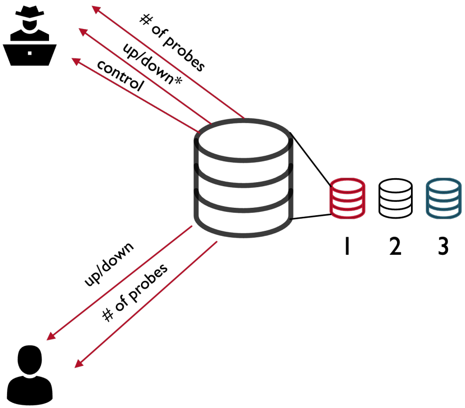
Observations

Attacker Knowledge

- Can only estimate when a server is compromised.
- Learns whether a server is up or down by probing.
- Knows who controls a server.
- Knows when a compromised server is reimaged.

Defender Knowledge

- Knows Which servers are down.
- Observes a probe with some probability.
- Unaware of a server being compromised.



Imperfect Observation

The state is only partially observable

The defender have observed only a few probes recently

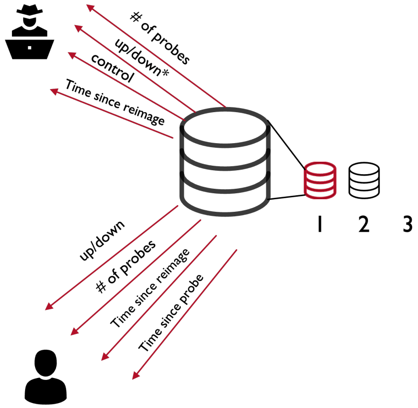
- *Is the server already compromised? The attacker is not probing it*
- *Is the server not compromised? There are few probes*

The attacker is unaware of an unprobed server

- *Has its progress been reset by reimaging?*
- *The previous probes are still in place?*

Including History

Improve RL with compact history representation



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Short-term Losses vs. Long-term Rewards

Short Term Loss

- **Defender**: Reimaging a server results in lower rewards while the server is offline
- **Attacker**: Probing incurs a cost

Long-term Gains

- **Defender**: A reimaged server will return the control to the defender
- **Attacker**: Continuous probes will compromise a server

The value of *discount factor* γ is important

Deep-Q-Learning for MTD

What Reinforcement Learning Algorithm to Choose?

- At each step, both agents decide on **a server** to probe or reimage
- Action space is **discrete**
- Observation features are **well-defined**
- *Feed Forward Neural Network* operates as a feature extractor
- **Deep-Q-Learning algorithm** can be used as the best-response oracle for both agents
- *Policy Space Response Oracles* will find the **Nash Equilibrium** of the MTD game

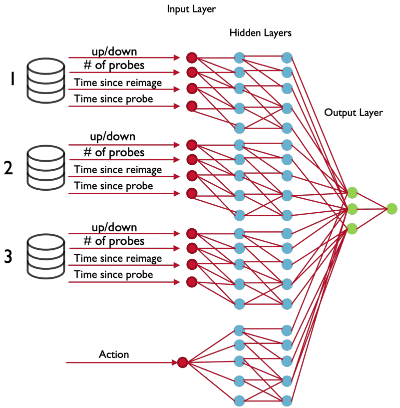


Figure 1: Defender NN Architecture

Evaluation

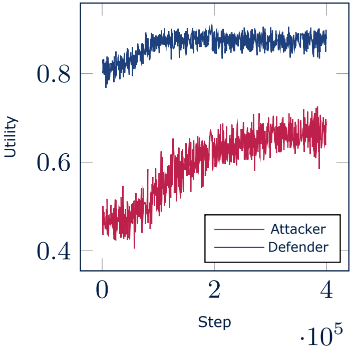


Figure 2: RL Training Curve for the first PSRO iteration

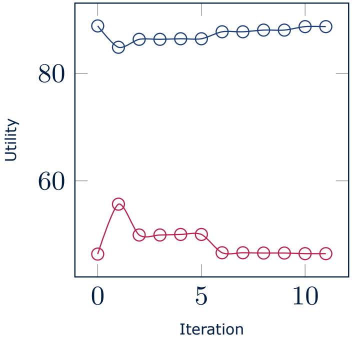


Figure 3: PSRO curve value after each iteration

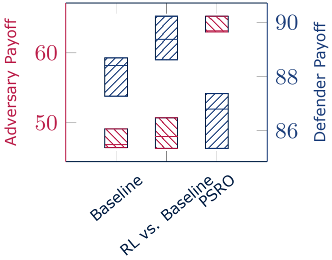
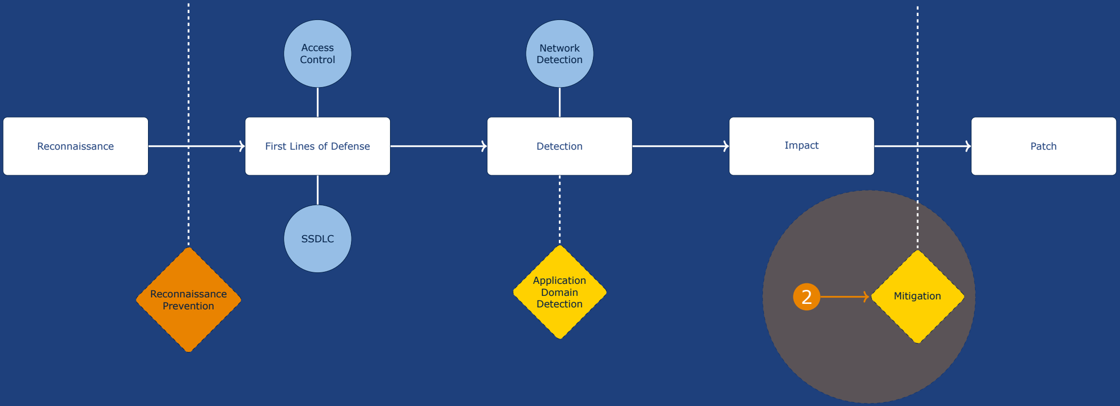


Figure 4: Episodic reward

Mitigation of False Data Injection in Industrial Control Systems



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Control System In Danger I

System Transitions

Differential equations dictate the transition rules

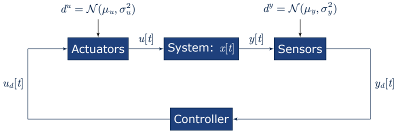
Attacker has Compromised Signals

- Will change sensor or actuator signals by a percentage to avoid detection

Presence of a Detector

A detector notifies the controller

[Giraldo et al., 2019, Paridari et al., 2018, Urbina et al., 2016]



From Associated Press

Control System In Danger II

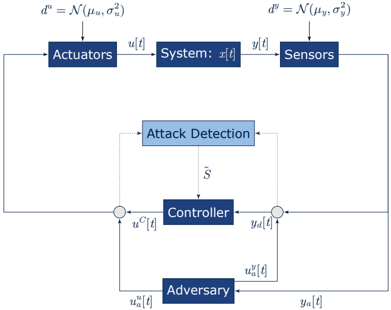
Observations, Actions, and Objectives

Attacker

- **Observes** sensor and actuator values
- **Perturbs** sensor and actuator values
- **Deviates** the system from nominal point

Defender

- **Observes** perturbed sensor signals
- **Observes** detection result
- **Decides** on control signals
- Minimize **worst-cast** deviations of nominal point



- **Attacker** gains reward by distance of system state to its nominal point $r_a = \|x - x_0\|$
- **Defender** gains reward by the negative of the distance $r_d = -\|x - x_0\|$

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Resilient Control through Reinforcement Learning

What Reinforcement Learning Algorithm to Choose?

- Action space is **continuous**
- Observation features are **well-defined**
- *Feed Forward Neural Network* operates as a feature extractor
- **Deep Deterministic Policy Gradients (DDPG)** can be used as the best-response oracle for both agents
- *Policy Space Response Oracles* will find the **Nash Equilibrium**, thus the resilient control policy

Evaluation I

Benchmark Systems

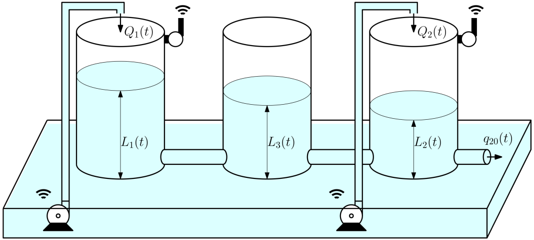


Figure 5: Schema of the **three tanks** system
Diagram from [Combita et al., 2019]

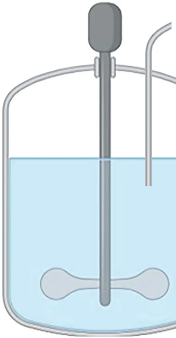


Figure 6: Schema of a **bioreactor**
Diagram from [Rahmatnejad et al., 2023]

Evaluations II

Bioreactor

Observation attacks cannot be tackled. There is no correlation between observation and reward.

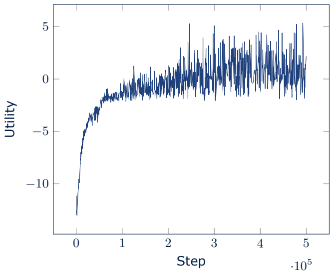


Figure 7: Learning curve of first iteration of DDPG in bioreactor

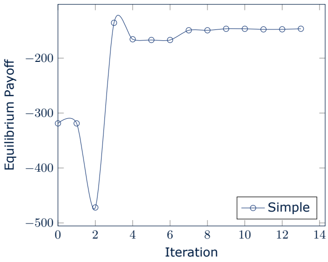


Figure 8: Episodic rewards in iterations of PSRO.

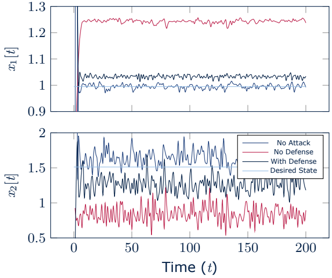


Figure 9: State Comparison

Evaluations II

Three Tanks

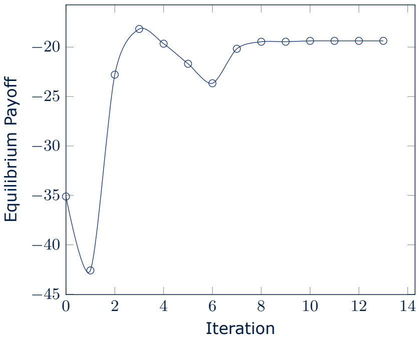


Figure 10: Episodic rewards in iterations of PSRO.

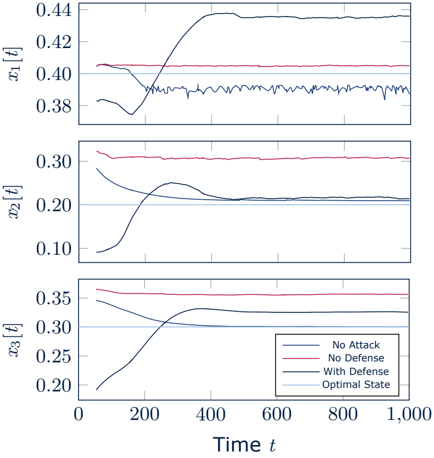
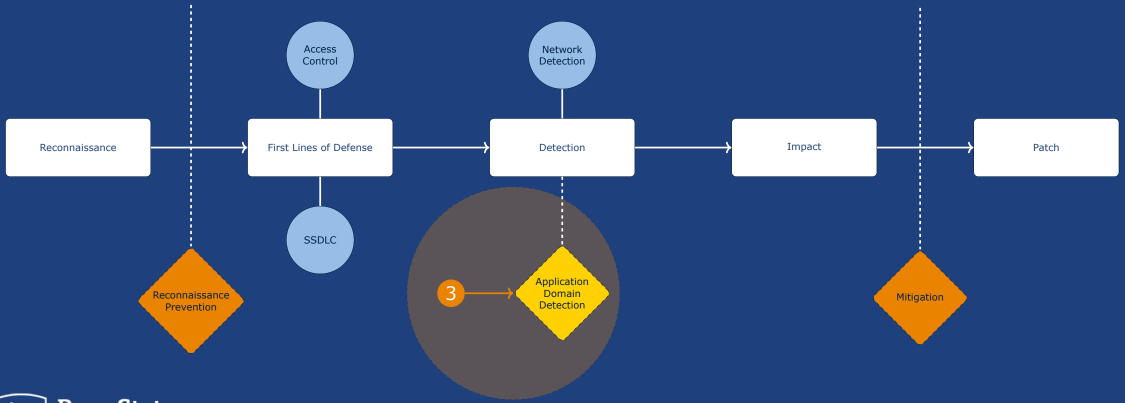


Figure 11: State Comparison

Detection of False Data Injection in Navigation Applications

[Eghtesad et al., 2024, Eghtesad et al., 2026]



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Attack on Navigation Applications I

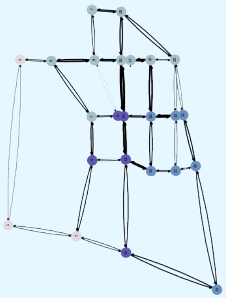
Transportation Model

- A bi-directional **graph** defines the transportation network's roads and intersections
  [Transportation Networks for Research Core Team, 2020]
- Each road has a given travel time: **Congestion Model**
- The more vehicles on a given road, the higher the travel time
- At each intersection, drivers look at the shortest path to destination by **the navigation application**

Threat Model

- Attacker has a budget to perturb perceived travel times
- Attacker perturbs perceived travel time
- Drivers take a longer path due to **perceived congestion**

Sioux Falls, SD



Attack on Navigation Applications II

Defense Model

- **Detection** of suspicious activity
- Detector raises an alarm
- Attack is defused if correctly detected

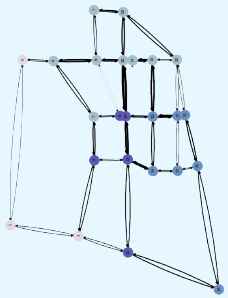
Observations

- Attacker has full observation of the network: vehicles, vehicle locations, driver destinations
- Detector only observes **reported** perceived travel time

Objectives

- **Attacker** gains reward by maximizing the total travel time
- **Detector** prevents increase in total travel time

Sioux Falls, SD



Attack detection through Reinforcement Learning

Let's try PSRO

- The attacker has full graph observation
- The attacker has a constrained continuous action
- The detector has a partial graph observation
- The detector takes a boolean action, either to raise an alarm or not

DDPG as attack oracle

- The attacker should output perturbations for thousands of city roads
- General-Purpose Reinforcement Learning algorithms are infeasible even for a small city
- It requires millions of samples collected from the environment
- We need a robust and **feasible** attack oracle

Hierarchical Multi-Agent Reinforcement Learning as Attack Oracle

Idea

- We can divide the network into **components** of smaller size.
- **Low-Level** RL agents are assigned to each component
- A **High-Level** RL agent coordinates the low-level agents

Why a high-level coordinator?

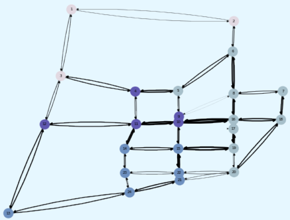
- *The total perturbations are restricted by a budget*
- *Component agents compete over the budget*

- The high-level agent allocates the perturbation budget to the component agents
- The low-level agents distribute given perturbation allocation to road links

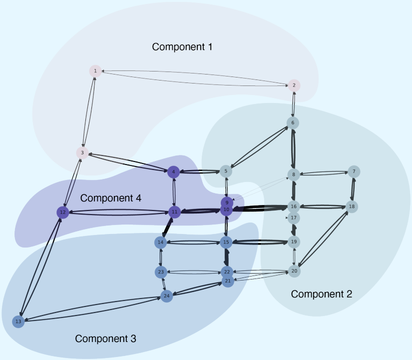
Network Decomposition

Decompose network based on **K-Means Clustering** by edge distance without congestion

Original Sioux Falls Network



Decomposed Sioux Falls Network



Hierarchical Approach I

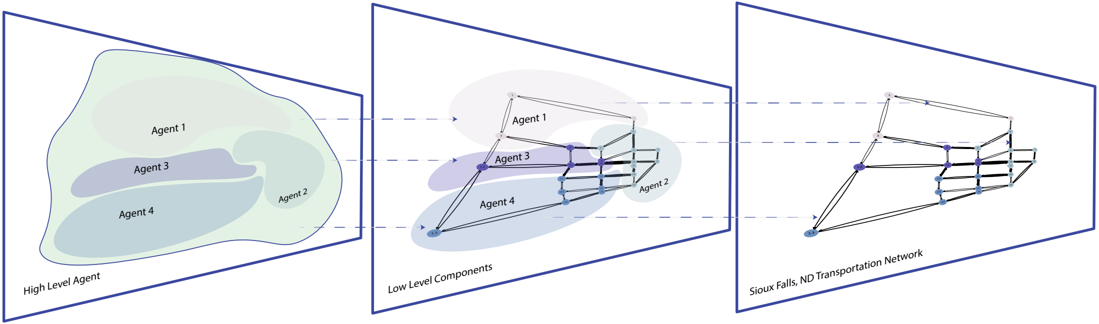
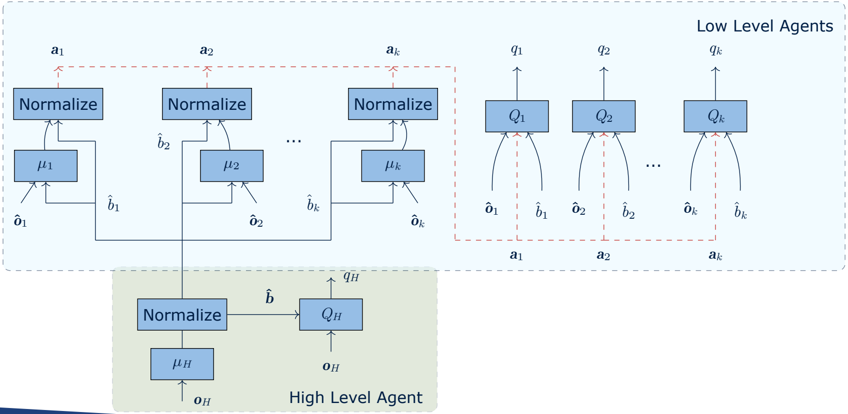


Figure 12: Hierarchical Multi-Agent Reinforcement Learning

Hierarchical Approach II

Reinforcement Learning Model



Hierarchical Approach III

Reinforcement Learning Observations

Each Low-Level observes 5 features per each edge e

1. Number of vehicles that are at an intersection with an unperturbed shortest path to the destination that passes through e
2. Number of vehicles currently on e
3. Number of vehicles that are at an intersection that will immediately take e as their shortest path without perturbation
4. Sum of remaining travel times of vehicles currently on edge e
5. Number of vehicles that are on an edge but will take e as the shortest path

High-Level observes

The sum of each feature per component as **summary**

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Hierarchical Approach IV

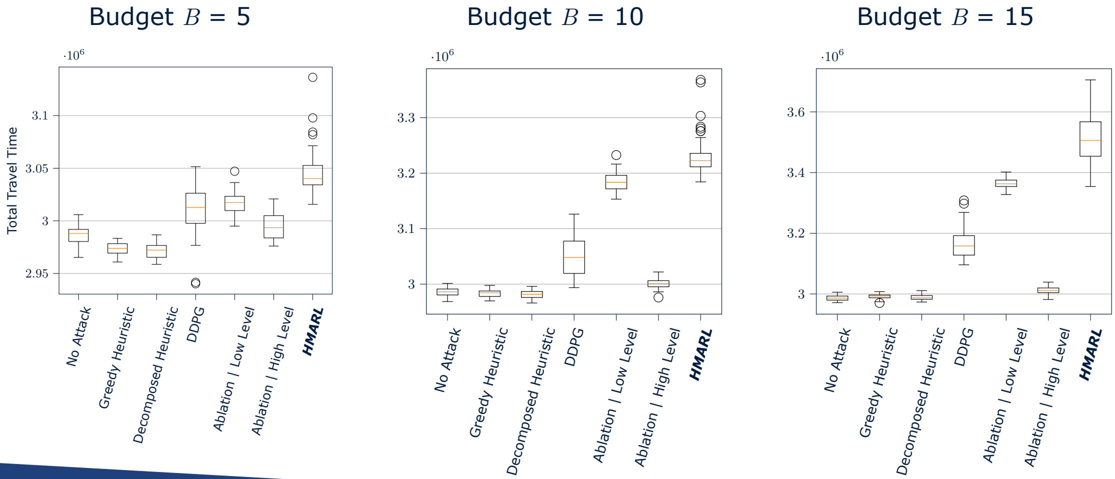
Rewards and Updating

Aim of the attacker is to maximize total travel time

- Low-level agents gain reward by the **number of vehicles in its component**
- High-level agent gain reward by the **total number of vehicles**
- This is equivalent to maximizing the total travel time.

Since the agents cooperate, all agents can be trained simultaneously.

Evaluation



Budget B = 5
Budget B = 10
Budget B = 15
$\cdot 10^6$
Total Travel Time
3.1
3.05
3
2.95
3.3
3.2
3.1
3
3.6
3.4
3.2
3
No Attack
Greedy Heuristic
Decomposed Heuristic
DDPG
Ablation / Low Level
Ablation / High Level
HMARL

Defense Mechanism

Observations

The defender observes the perturbed congestion times.

Action

The defender alerts administrators and technicians regarding anomalies in the traffic patterns. A **binary action** to raise an alarm or not.

Objective and Reward

- The reinforcement learning algorithm, automatically balances the cost of false positives and travel time of vehicles.
- The defender can be rewarded by negative of number of vehicles plus

Baselines

Attacker Baselines

Greedy

$$\mathbf{a}^t = \frac{\langle s_e^t : \forall e \in E \rangle}{\sum_{e \in E} s_e^t} \cdot B.$$

s_e^t: Number of vehicles passing through each edge e as their unperturbed shortest path to their destination [Eghtesad et al., 2024]

Gaussian

Divides the network into subcomponents i. Then:

$$a_e^t = \begin{cases} 0 & \text{if} \quad e \notin i \\ \sim \mathcal{N}(\hat{B} \cdot c_e, \frac{1}{10} \cdot c_e) & \text{otherwise} \end{cases} : \forall_{e \in E},$$

$\hat{B}$ is the budget [Yu et al., 2024]

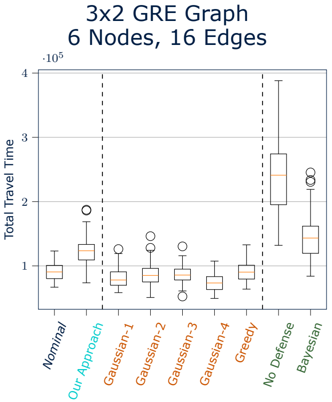
Defender Baselines

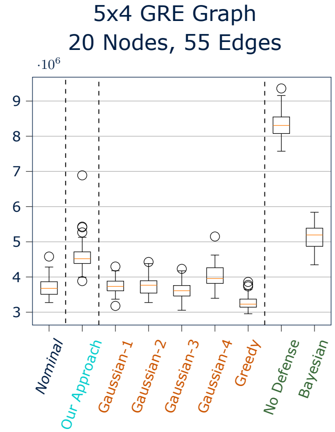
Bayesian

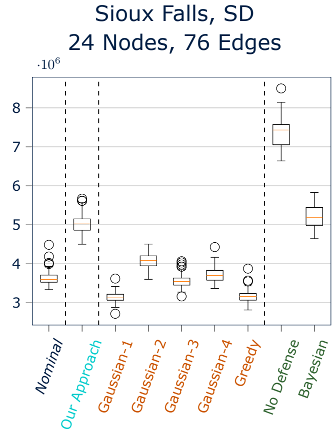
$$F_{\hat{w}} \sim \begin{cases} \text{Mean}(\hat{w}_e^t) = \mathbb{E}_x[\hat{w}_e^t] \\ \text{Cov}(\hat{w}_{ei}^{t_i}, \hat{w}_{ej}^{t_j}) = \mathbb{E}_x[(\hat{w}_{ei}^{t_i} - \mathbb{E}_x[\hat{w}_{ei}^{t_i}]) \\ \qquad \cdot (\hat{w}_{ej}^{t_j} - \mathbb{E}_x[\hat{w}_{ej}^{t_j}])] \end{cases}$$

Comparing likelihood of a history of observations $\hat{\mathbf{w}}_H$ according to this distribution with a threshold likelihood τ

Evaluations







Future Plans

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Defense Evasion and Alert Correlation

- Software products rely in **Intrusion Detection Systems (IDS)** to alert system admins of intrusions.
- A sophisticated attacker can evade detections using obfuscation methods.
- We can augment IDS with ML and RL based algorithms to **correlate** and **predict** alerts, providing safer and less noisy alerts to administrators.

Published Work

- **Taha Eghtesad**, Yevgeniy Vorobeychik, Aron Laszka; **Adversarial Deep Reinforcement Learning based Adaptive Moving Target Defense**; *Conference on Decision and Game Theory for Security (GameSec'20)*
- **Taha Eghtesad**, Sirui Li, Yevgeniy Vorobeychik, Aron Laszka; **Hierarchical Multi-Agent Reinforcement Learning for Assessing False-Data Injection Attacks on Trans-portation Networks**; *The 23rd International Conference on Autonomous Agents and Multiagent Systems (AAMAS'24)*;
- **Taha Eghtesad**, Yevgeniy Vorobeychik, Aron Laszka; **Adversarial Reinforcement Learning for Detecting False Data Injection Attacks in Vehicular Routing**; *17th ACM/IEEE International Conference on Cyber-Physical Systems (ICCPS'26).*

Other Published Articles

- O. Akgul, **T. Eghtesad**, et al; Bug Hunters' Perspectives on the Challenges and Benefits of the Bug Bounty Ecosystem; *USENIX Security'23*; **Core Ranking A***; **Distinguished Paper Award (Top %5)**
- O. Akgul, **T. Eghtesad**, et al; Exploring Challenges and Benefits of Bug-Bounty Programs; WSIW'20
- S. Eisele, **T. Eghtesad**, et al; Safe and Private Forward-Trading Platform for Transactive Microgrids; TCPS; Dec 2020
- S. Eisele, **T. Eghtesad**, et al; Blockchains for Transactive Energy Systems: Opportunities, Challenges, and Approaches; IEEE Computer; Sep 2020
- C. Barreto, **T. Eghtesad**, et al; Cyber-attacks and mitigation in blockchain based transactive energy systems; ICPS'20
- S. Eisele, **T. Eghtesad**, et al; Decentralized Computation Market for Stream Processing Applications; IC2E'22
- S. Eisele, **T. Eghtesad**, et al; Mechanisms for Outsourcing Computation via a Decentralized Market; DEBS'20

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Appendix

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Nash Equilibrium

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Policies and Strategies I

Pure Strategy

- A ***Strategy*** is a policy function that, given the current observation from the environment, produces an action to be taken by the agent.
- A ***Pure Strategy*** is a deterministic policy function.
- The ***Pure Strategy Set*** is the set of all possible pure strategies available to the player.
- A ***Pure Strategy Profile*** is a combination of all players strategy.
- The ***Utility Profile*** for player p is the amount of utility or reward the player p receives when players use a given strategy profile.

Pure Strategy

$$\pi(o) \mapsto a$$

Pure Strategy Set

$$\pi_p \in \Pi_p$$

Pure Strategy Profile

$$\boldsymbol{\pi} \leftarrow \{\pi_1, \pi_2, \cdots, \pi_p\}$$

Utility Profile

$$U_p(\boldsymbol{\pi})$$

Policies and Strategies II

Mixed Strategy

We need a mechanism to represent stochastic policies.

- A ***Mixed Strategy*** is a probability distribution over the player's pure strategy set.
- The ***Mixed Strategy Set*** is the set of all mixed strategies available to the player.
- The Mixed Strategy Utility Profile can be calculated using Pure Strategy Utility Profile.

Mixed Strategy Utility Profile

Utility from $\boldsymbol{\pi}$ times the probability that $\boldsymbol{\pi}$ occurs summed over all strategy profiles $\boldsymbol{\pi} \in \boldsymbol{\Pi}$.

Mixed Strategy

$$\boxed{\sigma_p(\pi_p)} \in [0,1]$$

$$\sum_{\pi_p}^{\Pi_p} \sigma_p(\pi_p) = 1$$

Mixed Strategy Set

$$\sigma_p \in \Sigma_p$$

Mixed Strategy Profile

$$\boldsymbol{\sigma} \leftarrow \{\sigma_1, \sigma_2, \cdots, \sigma_P\}$$

Utility Profile

$$U_p(\boldsymbol{\sigma}) = \sum_{\boldsymbol{\pi} \in \boldsymbol{\Pi}} \left(\prod_{p}^{P} \sigma_p(\pi_p) \right) \cdot U_p(\boldsymbol{\pi})$$

Policies and Strategies II

Best Response and Nash Equilibrium

- All players are **rational**. Thus, they pick a strategy that maximizes their utility.
- A ***Best-Response*** Mixed Strategy provides maximum utility for the player given the strategy of opponents:
- Assuming all players are using their best response, the strategy profile is a ***Mixed Strategy Nash Equilibrium*** (MSNE).

Best-Response Mixed Strategy

$$\boxed{\sigma_p^*}(\boldsymbol{\sigma_{-p}}) = \underset{\sigma_p \in \Sigma_p}{\operatorname{argmax}}\, U_p(\{\sigma_p, \boldsymbol{\sigma_{-p}}\})$$

Mixed Strategy Nash Equilibrium

$$\forall_{p \in P} \forall_{\sigma_p \in \Sigma_p} : \; U_p(\boldsymbol{\sigma^*}) \geq U_p(\{\sigma_p, \boldsymbol{\sigma^*_{-p}}\})$$

What is Mixed-Strategy Nash Equilibrium?

All players are playing with their best-response to all opponents' strategies, and neither player can increase their expected utility without having their opponents change their strategy.

Mixed Strategy Nash Equilibrium

MSNE Satisfies

$$u(\sigma_p, \sigma^*_{\bar{p}}) \leq u(\sigma^*_p, \sigma^*_{\bar{p}}) \leq u(\sigma^*_p, \sigma_{\bar{p}})$$

Independent Reinforcement Learning

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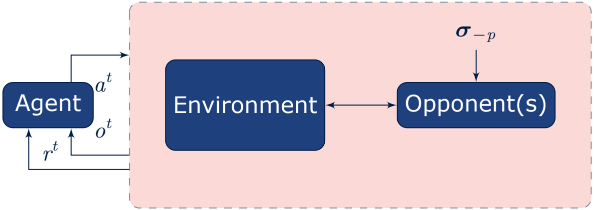
Reinforcement Learning Formalized I

Deep-Q-Learning I

- Extension of Q-Learning by [Mnih et al., 2015]
- Deterministic
- Off-Policy
- Value Iteration
- Action-Value Method
- Model-Free

RL Optimization Problem

$$\text{optimize } \pi(o^t) \mapsto a^t$$
$$\text{s.t. } \max \mathbb{E}\left[\sum_{\tau=0}^{\infty} \gamma^{\tau} \cdot r^{t+\tau} \,\middle|\, \pi, \boldsymbol{\sigma}_{-p}\right]$$



Reinforcement Learning Formalized II

Deep-Q-Learning II

The Parametric Q Function

$$Q^{\theta}(o^t, a^t) = r^t + \mathbb{E}\left[\sum_{\tau=1}^{\infty} \gamma^{\tau} \cdot r^{t+\tau} \,\middle|\, \pi\right]$$

Bellman Mean Squared Error

$$L(\theta) = \frac{1}{|X|} \sum_{x}^{X} \left(r_x^t + \gamma \cdot \operatorname*{argmax}_{a'} Q^{\theta}(o_x^{t+1}, a') - Q^{\theta}(o_x^t, a_x^t) \right)^2$$

Policy Function

$$a^t \leftarrow \pi(o^t) = \max_{a'} Q(o^t, a')$$

Experience

$$x = \langle o_x^t, a_x^t, o_x^{t+1}, r_x^t \rangle \in X \sim E$$

Reinforcement Learning Formalized III

Deep Deterministic Policy Gradients (DDPG)

$$\text{argmax}\ Q^{\theta} \leftrightarrows L(\theta)$$
$$\max\ Q^{\theta} \leftrightarrows \pi$$

Given that we have **Discrete** actions, we can enumerate them.

What if the action is not discrete? [Lillicrap et al., 2015]

Parameterized Policy Function

$$a^t \leftarrow \pi(o^t) = \boxed{\mu^{\Theta}(o^t)} \approx \underset{a'}{\text{argmax}}\ Q^{\theta}(o^t, a')$$

Policy Performance

$$J(\Theta) = \frac{1}{|X|} \sum_{x}^{X} Q^{\theta}\left(o_x^t, \mu^{\Theta}(o_x^t)\right)$$

Reinforcement Learning Formalized IV

Stochastic Policy Gradient I

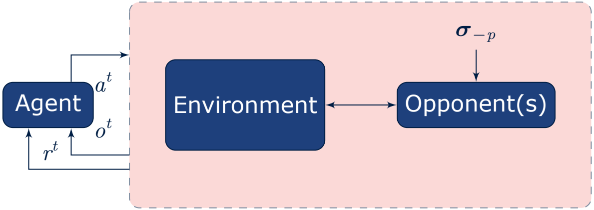
- Actor-Critic Methods
- Stochastic
- On-Policy
- Policy Iteration
- Value Method
- Model-Free

Improvements

SPG algorithm are improved with GAE [Schulman et al., 2016], TRPO [Schulman et al., 2015], and PPO [Schulman et al., 2017].

RL Optimization Problem

$$\text{optimize } \pi(o^t) \mapsto a^t$$
$$\text{s.t. } \max \mathbb{E}\left[\sum_{\tau=0}^{\infty} \gamma^\tau \cdot r^{t+\tau} \;\middle|\; \pi, \boldsymbol{\sigma}_{-p}\right]$$



Reinforcement Learning Formalized V

Stochastic Policy Gradient II

The idea of SPG is to update policy to increase the probability of actions with a positive advantage.

Stochastic Actor

$$a^t \sim \pi^{\Theta}(o^t)$$

Value Function

$$V_\pi(o^t) = \mathbb{E}\left[\sum_{\tau=0}^{\infty} \gamma^\tau \cdot r^{t+\tau} \,\middle|\, \pi\right]$$

Advantage

$$A(o^t, a^t) = r^t + \gamma \cdot V_\pi(o^{t+1}) - V(o^t)$$

Policy Performance

$$J(\Theta) = \frac{1}{|X|}\sum_{x}^{X} \boxed{\log \pi^{\Theta}(a_x^t|o_x^t)} \cdot A(o_x^t, a_x^t)$$

Value Function Loss

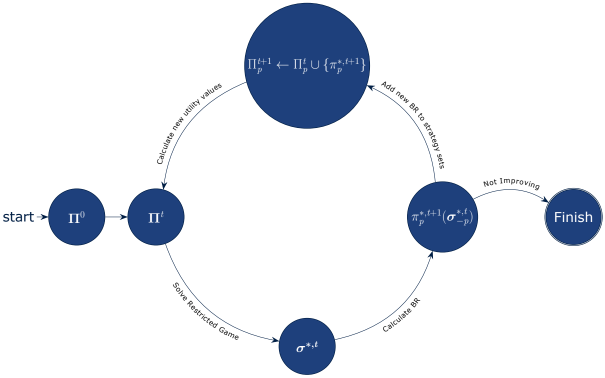
$$L(\theta) = \frac{1}{|X|}\sum_{x}^{X} \left(r_x^t + \gamma \cdot V_\pi(o_x^{t+1}) - V_\pi(o_x^t)\right)^2$$

Experience

$$x = \langle o_x^t, a_x^t, o_x^{t+1}, r_x^t \rangle \in X \sim \mathfrak{T}$$

Policy Space Response Oracles

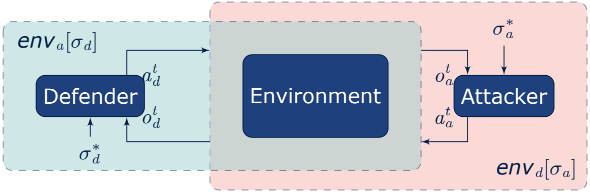
Policy Space Response Oracles



Require: Initial strategy sets $\mathbf{\Pi}$;
Compute *Expected Utilities* $U^{\boldsymbol{\pi}}$ for each strategy profile $\boldsymbol{\pi} \in \mathbf{\Pi}$;
Compute MSNE of $\mathbf{\Pi}$ as $\boldsymbol{\sigma}^*$;
for *many epochs* **do**
  for *each player* p **do**
    for *many episodes* **do**
      Sample $\boldsymbol{\pi}^*_{-p} \sim \boldsymbol{\sigma}^*_{-p}$;
      Train $\pi_p^+(\boldsymbol{\pi}_{-p})$ using InRL;
    end
    $\Pi_p \leftarrow \Pi_p \cup \{\pi_p^+\}$;
  end
  Compute $U^{\boldsymbol{\pi}}$ for new strategies ;
  Compute MSNE of $\mathbf{\Pi}$ as $\boldsymbol{\sigma}^*$;
end
Output current solution $\boldsymbol{\sigma}^*$;

PSRO algorithm [Lanctot et al., 2017] based on Double Oracles [McMahan et al., 2003].

Policy Space Response Oracles



Result: set of pure policies Π^a and Π^d
$\Pi_a \leftarrow$ attacker heuristics;
$\Pi_d \leftarrow$ defener heuristics;
while $U_p(\sigma_p, \sigma_{-p})$ *not converged* **do**
  $\sigma_a, \sigma_d \leftarrow$ `solve_MSNE`(Π_a, Π_d);
  $\theta \leftarrow$ `random`;
  $\pi_a^+ \leftarrow$ `train`$(T \cdot N_e, env_a[\sigma_d], \theta)$;
  $\Pi_a \leftarrow \Pi_a \cup \pi_a^+$;
  assess π_a^+;
  $\sigma_a, \sigma_d \leftarrow$ solve_MSNE(Π_a, Π_d);
  $\theta \leftarrow$ `random`;
  $\pi_d^+ \leftarrow$ `train`$(T \cdot N_e, env_d[\sigma_a], \theta)$;
  $\Pi_d \leftarrow \Pi_d \cup \pi_d^+$;
  assess π_d^+;
end